**2.2 Derivation of the Transfer Function:**

The Transfer Function *G(s)* is produced using the formula:

For which P(s) is the output and Q(s) is the input of the system. For this case, the system dynamics is depicted by the following differential equations:

Within the system, the following symbols, , , and denote pitch angle, angle of attack, pitch rate and the deflection angle of the elevators respectively. Which harbour crucial roles in regulating the pitch control of the aircraft. To obtain the transfer function of the system, the Laplace transform method is applied on each of the differential equations in which zero initial conditions are assumed. The transfer function will utilise the aforementioned results and take the form of:

The Laplace transformations of each of the differential equations are as follows:

The final Transfer Function after further algebraic manipulation to find in terms of is:

Where ), pitch angle, represents the output and , deflection angle of the elevators, represents the input.

The final form of the Transfer Function is the concise representation that encompasses the dynamics of the system. By representing the relationship between the system’s input and output signals in the frequency domain. Hence serving as a mathematical model exhibiting the essential characteristics of the system’s behaviour. This Transfer Function will be the used as the basis for the controller design that will be further investigated as this report progresses.

**2.3 Preliminary Analysis**

To illustrate the performance of the aircraft system itself, analysis will be carried out to study the properties of the open-loop dynamics. For this to be achieved, poles and zeros, stability and system responses will be examined.

**2.3.1 Stability Analysis**

This subsection focuses on the poles and zeroes of the Transfer Function. Upon examination of *G(s)*, it is observed that there is one zero located at the origin, this being s=0, while there are two poles located within the roots of the quadratic polynomial on the denominator of the Transfer Function at, These roots on the denominator, Q, are the poles of *F*, and the roots of P are the zeroes of *F*. This zero being at the origin connotes that the output of the system,), is directly proportional to its input, with no additional dynamics.

**2.3.2 Pole Location Analysis**

The next aspect to be examined with regards to stability is the poles position in the complex plane. For the Transfer Function calculated, the poles are located at = -0.3695+0.88570297j and = -0.3695-0.88570297j while the zero is at .These poles having negative real parts indicates stability, as the poles are in the left-half of the complex plane exhibiting the system’s natural reaction to lean towards its equilibrium state over time. Indicating that in instances in which the system would experience external disturbances, in this case, turbulence, wind gusts and structural flexibility equilibrium restoration occurs. Hence, the negative real part suggests that the response diminishes exponentially over time, approaching zero. Exhibiting stability within the system, as the disturbances affecting its performance would lessen within time, to ensure its robust performance and resilience. Additionally, the presence of a zero at affects the system’s responses by cancelling out certain dynamics contributing to the transient response.

**2.3.3 System Response Analysis**

**Bode Plot**

**A graph of a function

Description automatically generated with medium confidence**

Figure 1. Bode plot measuring Magnitude (dB) against Frequency (rad/sec) and Phase (deg.) against Frequency (rad/sec). The Bode plot exhibits how the output signal (pitch angle) changes with frequency. To show the gain as a function of frequency. The phase plot, displays the phase shift initiated by the system at different frequencies, indicating the phase delay or advance of the output signal relative to the input signal as a function of frequency.

The Bode plot represents the overall system response, by providing comprehensive insight into the frequency-domain behaviour of the open-loop system. In which information about the magnitude of the magnitude and phase response can also be observed. With regards to the magnitude curve, it indicates a system with a low frequency gain of approximately 0 dB, meaning that low-frequency inputs will pass through the system without any attenuation. As the frequency increases the magnitude curve begins to roll off at a rate of -20dB / decade, which shows aspects of a second-order system, which attenuates or filters out high-frequency components. Within the same time range, the phase curve begins at 0 degrees then continuously decreases as the frequency increases, then reaching a maximum phase lag of -135 degrees as these higher frequencies. The omission of any resonant peaks or abrupt changes in the Bode Plot would suggest stability within the system, indicating a second order system without additional complex poles or zeroes. Although, the relatively low roll off rate and prominent phase lag at high frequencies resulted in the observed oscillatory and slow response attributes within the examined time domain. From the overall shape and characteristics seen on the graph, the aircraft pitch control system is approximated to be a second-order system. The nature of this system has implications for its transient response, such as oscillations and the natural frequency.

**Step response**

A graph showing a line graph

Description automatically generated with medium confidence

Figure 2. A graph of the pitch angle (rad.) against Time (seconds) the Transfer Function of the control system. The step response is the time-domain reaction of the system to sudden instantaneous changes in the input. In this case a step input. Providing insight into the information about the system’s transient behaviour and performance characteristics.

From the observations made with of from the step response, extensive insights into the behaviour following a sudden and sustained change is given. As it shows a smooth and monotonic increase, which eventually approaches a steady-state value, with no overshoots or oscillations. The response curve has a gentle slope which indicates a slow rise time and a gradual approach towards the steady-state value, which is at 0.6 radians, representing the final pitch angle achieved by the system in response to the step input. The rise time, which is the time taken for the response to rise towards the steady-state value is 3 seconds. The slow rise time and lack of full oscillations suggest that the system has an overdamped response, which leads to a longer settling time before reaching the steady-state value. Therefore, the settling time to reach the steady-state value appears to be relatively long, as seen from the graph exhibiting the time to be around 10 seconds beyond the visible scale. Despite the absence of an overshoot in the response, the lack of any rapidity implies a lagging response, which poses challenges in aircraft applications, therefore prompting the requirement for a swift and responsive control system to counteract this flaw.

**Impulse Response**

A graph showing a line graph

Description automatically generated

Figure 3. The pitch angle (rad.) against Time (sec) showing the system’s reaction upon undergoing a brief impulse or spike input. Which are sudden disturbances, this aids in assessing the system’s stability and responsiveness.

The results seen from the impulse response exhibits oscillatory behaviour, with these oscillations gradually decreasing in amplitude over the given time period. From the shape of the graph, it shows that the oscillations do not diverge or grow unbounded, which indicates that the open loop is stable. However, the time period these oscillations exhibit have a relatively long settling time which suggests that system has low damping. The peak of the impulse response is seen to occur around 0.5 seconds, with a maximum amplitude reaching 0.17 radians. This response implies a damped oscillatory behaviour with the first peak-to-peak amplitude being the largest and the following peak-to-peak amplitudes gradually decreasing. The period of the oscillations is 3.5 seconds, giving insight into the natural frequency of the system, these oscillations are seen to persist for a relatively long time, as they are visible after 15 seconds. This slow settling time indicates potential for sustained oscillations, however this prolonged oscillatory behaviour is undesirable for a control system, as the aircraft requires a quick and stable response.

From the analysis of the transfer function, in terms of its response, such as bode plots, impulse and step response, it is apparent that the necessity for a controller is crucial. As the impulse and step response graphs indicate that the open-loop system is stable, due to the characteristics of the responses not diverging or growing unbounded. For example, the impulse response shows damped oscillations, indicating an underdamped system. This behaviour leads to overshoots and oscillations in the system’s response to input signals. While the step response conveys a slow rise time and settling time, which are delayed responses to input changes. Therefore, a controller needs to be introduced, to be able to include additional damping and quicker response times. It would be created to minimise any oscillations, while reducing settling times and providing a desirable balance between responsiveness and stability.